BRL

REPORT NO. 287
JULY 1942

491854

THE RELATION BETWEEN THE SKIN FRICTION DRAG AND THE SPIN REDUCING TORQUE

A. C. Charters
R. H. Kent

BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

Ba~~llistic Resea~~rch
~~Laboratory Report No. 287~~

[redacted]

(11) 4 Jul[redacted]42,

(12) 6p.

(6) THE RELATION BETWEEN THE SKIN FRICTION DRAG AND THE SPIN REDUCING TORQUE,

(10) A. C. Charters and R. H. Kent.

[redacted]

(14) BRL-287

On certain simplifying assumptions, a relation between the skin friction drag and the spin reducing torque or axial couple is deduced. On the basis of measured values of the axial couple coefficient the skin friction drag coefficient is deduced and compared with the measured total drag. a method is derived for utilizing

It occurred to ~~one of us* that~~ the measured values of the axial couple coefficient ~~should afford a method of estimating~~ to estimate, the skin friction drag coefficient. The purpose of this report is to derive a method of making this estimate.

Consider a shell of caliber, d, moving nose on into the air with a velocity, v, and a velocity of rotation, ω. We make the assumption that the skin friction drag on an element dS of the surface of the shell is

$$\rho\, f(z)\, u^2\, dS$$

* A. C. Charters.

in magnitude, where u is the velocity of the element with respect to the atmosphere, ρ is the density of the undisturbed air and $f(z)$ is a function of z, the distance of the element from the base of the shell. The direction of the skin friction drag on the element dS is taken to be opposite to the direction of motion of the element dS.

If the radius of the shell at the section on which dS lies is r, the relative velocity u is evidently the vector sum of v and $r\omega$ or

$$\sqrt{v^2 + r^2\,\omega^2} = u.$$

The component of this force parallel to the axis of the shell is obviously

$$\rho\,\frac{v}{u}\,f(z)\;u^2 dS = \rho f(z)\;u\;v\;dS$$

and the total skin friction drag on these assumptions is

$$\int \rho\;f(z)\;uv\;dS = \rho\;\bar{u}\;v \int f(z)\;dS \qquad (1)$$

if $\bar{u}$ is an appropriate average value of u.

The tangential component of the skin friction force is evidently

$$\rho\,\frac{r\omega}{u}\,f(z)\;u^2\;dS = \rho\;f(z)\;r\;\omega\;u\;dS.$$

The torque produced by this force acting on the element dS is

$$\rho\;f(z)r^2\;\omega\;u\;dS$$

and the total torque is

$$\int \rho\;f(z)\;r^2\omega u\;dS = \rho\omega\bar{u}' \int f(z)r^2\;dS \qquad (2)$$

if $\bar{u}'$ is another appropriate average value of u.

It is apparent that to evaluate (1) one needs to know the value of $\int f(z)dS$.

If $\overline{r^2}$ represents a certain average value of r^2 and the torque is known, $\int f(z)\;dS$ is determined by the relation

$$\int f(z)\, dS = \frac{(\text{torque})}{\overline{r}^2\, \rho\omega\, \overline{u}'}$$

and the skin friction drag is given by

$$\rho\, \overline{u}\, v \int f(z)\, dS = \frac{v(\text{torque})\overline{u}}{\overline{r}^2\, \omega\, \overline{u}'} . \tag{3}$$

We accordingly take the case of a cylindrical shell, like a proof slug, and neglect the skin friction drag on the nose and the base. Under these conditions

$$\overline{u} = \overline{u}' = \sqrt{v^2 + \frac{d^2\omega^2}{4}} ,$$

$\overline{r} = d/2$, and the skin friction drag is

$$\frac{4\, v\, (\text{torque})}{d^2\, \omega} . \tag{4}$$

According to Ballistic Research Laboratory Report No. 154, the torque designated by M_s has been represented by

$$M_s = C_A\, \rho\, d^4\, \omega\, v, \tag{5}$$

where C_A is the axial couple coefficient. Practical units are used as follows:

Quantity	Units
M_s	lb ft²/sec²
ρ	ratio of air density to normal
d	in
ω	rad/sec
v	ft/sec

If self consistent units are used we have

$$M_s = K_A\, \rho\, d^4\, \omega\, v, \tag{6}$$

where K_A the axial couple coefficient is dimensionless. It may be shown that

$$K_A = \frac{12^4}{.0751} C_A = 2.76 \times 10^5 \ C_A.$$

From (4) and (6)

$$D_F = \frac{4v \ \rho \ d^4 \omega v \ K_A}{d^2 \omega} = 4K_A \rho \ d^2 v^2 \qquad (7)$$

where D_F is the skin friction drag. If the skin friction drag coefficient is designated by K_{DF} so that

$$D_F = K_{DF} \ \rho \ d^2 \ v^2,$$

then from (7)

$$K_{DF} \ \rho \ d^2 v^2 = 4K_A \ \rho \ d^2 v^2 \quad \text{or,} \quad K_{DF} = 4K_A.$$

According to Captain Sterne's analysis of Dr. Van Allen's results the value of C_A for the 3" shell M42 is 2.12×10-8 from which the value of K_A is found to be 5.85×10-3. The corresponding value of K_{DF} is therefore $4 \times 5.85 \times 10$-3 = .0234.*

The drag coefficient of this shell is approximately .153 at a velocity of 2000 ft/sec. Hence the ratio of the skin friction drag to the total drag is, on these assumptions

$$\frac{.0234}{.153} = .153.$$

If the skin friction drag is evaluated by (3) instead of (4), in other words, if r^2 is substituted for $d^2/4$, we have

$$K_{DF} = \frac{d^2}{r^2} \frac{\overline{u}}{\overline{u'}} \ K_A,$$

and the ratio $\frac{K_{DF}}{K_D}$ may be larger than the value given, .153.

* The customary aerodynamic expression for the skin friction drag is:

$$D_F = C_{DF} \ \rho \ S \ v^2$$

where S is the surface of the projectile. For the 3" shell M42, the calculated S is 103.5 in^2, and S/d^2 is 11.50. Hence,

$$C_{DF} = .00203, \quad K_A = .000{,}508 \ S/d^2.$$

If S' represents the surface of the projectile, exclusive of the base, S' is 96.5 in^2 and S'/d^2 is 10.72 for the 3" shell M42. Then,

$$C'_{DF} = .00218, \quad K_A = .000{,}545 \ S'/d^2.$$

von Kármán[1] and Moore give 11% as an estimated value of this ratio.

It follows from the argument of page 2 equation (2) that in place of (5) the axial couple could be expressed more accurately in the form

$$M_s = K_A \rho d^4 \omega \bar{u}'$$

and for the cylindrical shell this would become

$$M_s = K_A \rho d^4 \omega \sqrt{v^2 + d^2 \omega^2/4}\ ^*.$$

A. C. Charters

A. C. Charters

R. H. Kent

R. H. Kent

[1] von Kármán and Moore, Trans. A.S.M.E., June 1932.
* This form was proposed independently by J. A. Van Allen in a letter dated June 25th, received while this report was being written.

RE[redacted]

TITLE: Relation between the Skin Friction Drag and the Spin Reducing Torque

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AUTHOR(S): Kent, R.H.; Charters, A.C.
ORIGINATING AGENCY: Aberdeen Proving Ground, Ballistic Research Lab., Md.
PUBLISHED BY: (Same)

ATI-40334
REVISION (None)
ORIG. AGENCY NO. R-287
PUBLISHING AGENCY NO. (Same)

DATE	DOC. CLASS.	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
July '42	[redacted]	U.S.	Eng.	5	(None)

ABSTRACT:

EO 10501 dd 5 NOV 1953

A method is derived for making an estimate of the skin friction drag coefficient for projectiles. On certain simplifying assumptions, a relation between the skin friction drag and the spin reducing torque or axial couple is deduced. On the basis of measured values of the axial couple coefficient the skin friction drag coefficient is deduced and compared with the measured total drag.

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DIVISION: Ordnance and Armament (22)
SECTION: Ballistics (12)

22-4,6
4,6

ATI SHEET NO.: R-22-12-34

SUBJECT HEADINGS: Projectiles,- Drag (75415.6); Skin friction - Theory (86798.5)